Jun 10

12

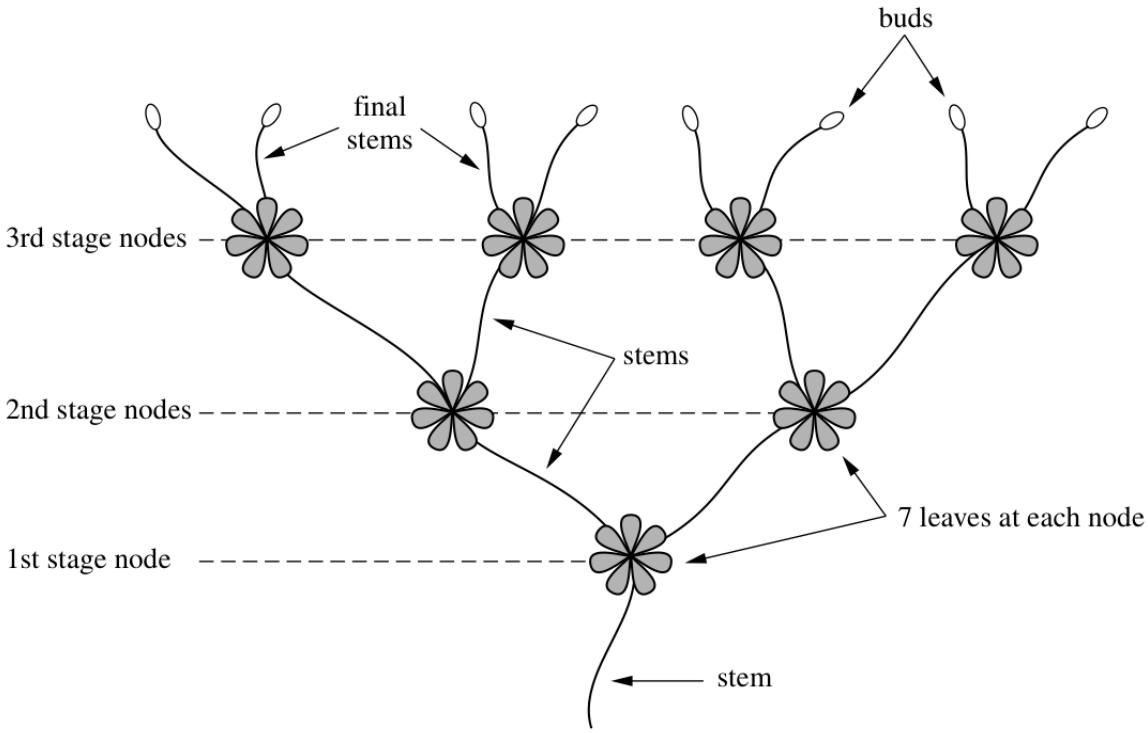

Fig. 12

A branching plant has stems, nodes, leaves and buds.

- There are 7 leaves at each node.
- From each node, 2 new stems grow.
- At the end of each final stem, there is a bud.

Fig. 12 shows one such plant with 3 stages of nodes. It has 15 stems, 7 nodes, 49 leaves and 8 buds.

(i) One of these plants has 10 stages of nodes.

(*A*) How many buds does it have? **[2]**

(*B*) How many stems does it have? **[2]**

(ii) (*A*) Show that the number of leaves on one of these plants with n stages of nodes is

$$7(2^n - 1).$$ **[2]**

Jan 11

5 The second term of a geometric sequence is 6 and the fifth term is −48.

Find the tenth term of the sequence.

Find also, in simplified form, an expression for the sum of the first n terms of this sequence. **[5]**

Jan 12

9 A geometric progression has a positive common ratio. Its first three terms are 32, b and 12.5.

Find the value of b and find also the sum of the first 15 terms of the progression. **[5]**

Jun 12

11 A geometric progression has first term a and common ratio r. The second term is 6 and the sum to infinity is 25.

(i) Write down two equations in a and r. Show that one possible value of a is 10 and find the other possible value of a. Write down the corresponding values of r. **[7]**

(ii) Show that the ratio of the nth terms of the two geometric progressions found in part (i) can be written as $2^{n-2} : 3^{n-2}$. **[3]**

Jan 13

(ii) A geometric progression has first term a and common ratio r, with $r \neq \pm 1$. The sum of its first two terms is 25 and the sum of its first four terms is 250.

Use the formula for the sum of a geometric progression to show that $\frac{r^4 - 1}{r^2 - 1} = 10$ and hence or otherwise find algebraically the possible values of r and the corresponding values of a. **[5]**

Let x = r squared to form a quadratic equation in x for the last part of this question

Jun 13

6 S is the sum to infinity of a geometric progression with first term a and common ratio r.

(i) Another geometric progression has first term $2a$ and common ratio r. Express the sum to infinity of this progression in terms of S. **[1]**

(ii) A third geometric progression has first term a and common ratio r^2. Express, in its simplest form, the sum to infinity of this progression in terms of S and r. **[2]**

Jun 14

7 The second term of a geometric progression is 24. The sum to infinity of this progression is 150. Write down two equations in a and r, where a is the first term and r is the common ratio. Solve your equations to find the possible values of a and r. **[5]**

Jun 15

11 Jill has 3 daughters and no sons. They are generation 1 of Jill's descendants.

Each of her daughters has 3 daughters and no sons. Jill's 9 granddaughters are generation 2 of her descendants. Each of her granddaughters has 3 daughters and no sons; they are descendant generation 3.

Jill decides to investigate what would happen if this pattern continues, with each descendant having 3 daughters and no sons.

(i) How many of Jill's descendants would there be in generation 8? **[2]**

(ii) How many of Jill's descendants would there be altogether in the first 15 generations? **[3]**

These last two questions concern both APs and GPs

Jun 16

3 An arithmetic progression (AP) and a geometric progression (GP) have the same first and fourth terms as each other. The first term of both is 1.5 and the fourth term of both is 12. Calculate the difference between the tenth terms of the AP and the GP. **[5]**

Jun 17

11 A firm takes on two new employees, Arif and Bettina.

- Arif starts on an annual salary of £30 000, and his salary increases by £1000 each year after that.
- Bettina starts on an annual salary of £25 000, and her salary then increases by 5% each year after that. (So, for example, Bettina's salary in year 3 is 5% greater than her salary in year 2.)

(i) Show that Arif earns more than Bettina in year 10 of their employment, but Arif earns less than Bettina in year 11. **[4]**

(ii) Show that the total amounts earned by each of Arif and Bettina during their employment up to the end of year 17, correct to the nearest £100, are equal. **[4]**