Transition-Arithmetic Progressions Self-Assessed Exercise Solutions

Jan 05

6 A sequence is given by

$$a_1 = 4,$$

$$a_{r+1} = a_r + 3.$$

Write down the first 4 terms of this sequence.

Find the sum of the first 100 terms of the sequence. [5]

$a_1 = 4$, $a_2 = 7$, $a_3 = 10$, $a_4 = 13$

AP $a = 4$, $d = 3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{100} = \frac{100}{2}[8 + 99 \times 3] = 15,250$

Jun 05

2 The nth term of an arithmetic progression is $6 + 5n$. Find the sum of the first 20 terms. [4]

1st term $= 6 + 5(1) = 11$

2nd term $= 6 + 5(2) = 16$

AP $a = 11$, $d = 5$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{20} = \frac{20}{2}[22 + 19 \times 5] = 1170$

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12 **(i)** Granny gives Simon £5 on his 1st birthday. On each successive birthday, she gives him £2 more than she did the previous year.

(*A*) How much does she give him on his 10th birthday? [2]

(*B*) How old is he when she gives him £51? [2]

(*C*) How much has she given him **in total** when he has had his 20th birthday present? [2]

AP $a = 5$, $d = 2$

A) 10^{th} term $= a + 9d = 5 + 9 \times 2 = £23$

B) n^{th} term $= a + (n-1)d$

$= 5 + 2(n-1) = 51$

$5 + 2n - 2 = 51$

$2n = 48$

$n = 24$

24 years old

C) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[10 + 19 \times 2] = £480$

6 A sequence is given by the following.

$$u_1 = 3$$
$$u_{n+1} = u_n + 5$$

(i) Write down the first 4 terms of this sequence. [1]

(ii) Find the sum of the 51st to the 100th terms, inclusive, of the sequence. [4]

i) $u_1 = 5,\quad u_2 = 9,\quad u_3 = 13,\quad u_4 = 17$

ii) AP $a = 5,\ d = 4$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

We require $S_{100} - S_{50}$

$$S_{100} = \frac{100}{2}\left[10 + 99\times 4\right] = 20,300$$

$$S_{50} = \frac{50}{2}\left[10 + 49\times 4\right] = 5,150$$

$$S_{100} - S_{50} = 15,150$$

8 The 7th term of an arithmetic progression is 6. The sum of the first 10 terms of the progression is 30.

Find the 5th term of the progression. [5]

7^{th} term $\quad a + 6d = 6 \quad (1)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + 9d] = 30$

$10a + 45d = 30 \quad (2)$

(1) x 10 $\quad 10a + 60d = 60 \quad (3)$

(3) − (2) $\quad 15d = 30$

$d = 2$

Sub for d in (1) $\quad a + 6(2) = 6$

$a = 6 - 12$

$a = -6$

5^{th} term $= a + 4d$

$= -6 + 4(2)$

$= 2$

Jun 07

11 **(a)** André is playing a game where he makes piles of counters. He puts 3 counters in the first pile. Each successive pile he makes has 2 more counters in it than the previous one.

(i) How many counters are there in his sixth pile? [1]

(ii) André makes ten piles of counters. How many counters has he used altogether? [2]

AP $a = 3$, $d = 2$

i) 6^{th} term $= a + 5d = 3 + 5(2) = 13$

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ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[6 + 9(2)] = 120$

Jun 08

8 The 11th term of an arithmetic progression is 1. The sum of the first 10 terms is 120. Find the 4th term. [5]

11^{th} term $= a + 10d = 1$ (1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + 9d] = 120$

$10a + 45d = 120$ (2)

(1) x 10 $10a + 100d = 10$ (3)

③ − ②

$55d = -110$

$d = -\frac{110}{55}$

$d = -2$

Sub for d in ①

$a + 10(-2) = 1$

$a = 21$

4^{th} term $= a + 3d$

$= 21 + 3(-2)$

$= 15$

Jan 09

6 An arithmetic progression has first term 7 and third term 12.

(i) Find the 20th term of this progression. **[2]**

(ii) Find the sum of the 21st to the 50th terms inclusive of this progression. **[3]**

1st term $a = 7$ ①

3rd term $a + 2d = 12$ ②

② − ①

$2d = 5$

$d = \frac{5}{2}$

i) 20^{th} term $= a + 19d$

$= 7 + 19 \times 2.5$

$= 54.5$

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ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

We require $S_{50} - S_{20}$

$S_{50} = \frac{50}{2}[14 + 49 \times 2.5] = 3412.5$

$S_{20} = \frac{20}{2}[14 + 19 \times 2.5] = 615$

$S_{50} - S_{20} = 2797.5$

Jun 09

11 **(i)** In a 'Make Ten' quiz game, contestants get £10 for answering the first question correctly, then a further £20 for the second question, then a further £30 for the third, and so on, until they get a question wrong and are out of the game.

(*A*) Haroon answers six questions correctly. Show that he receives a total of £210. **[1]**

(*B*) State, in a simple form, a formula for the total amount received by a contestant who answers n questions correctly.

Hence find the value of n for a contestant who receives £10 350 from this game. **[4]**

A) $10 + 20 + 30 + 40 + 50 + 60 = £210$

B) $S_n = \frac{n}{2}[2a + (n-1)d]$

AP $a = 10$, $d = 10$

$$S_n = \frac{n}{2}\left[20 + 10(n-1)\right]$$

$$S_n = \frac{n}{2}\left[10 + 10n\right]$$

$$S_n = 5n(1+n)$$

If $S_n = 10{,}350 = 5n(1+n)$

$$5n^2 + 5n - 10350 = 0$$

$$n^2 + n - 2070 = 0$$

$$(n+46)(n-45) = 0$$

$n = -46$ or $n = 45$

Jan 10

6 **(i)** Find the 51st term of the sequence given by

$$u_1 = 5,$$
$$u_{n+1} = u_n + 4.$$

[3]

$u_1 = 5$, $u_2 = 9$ AP $a = 5$, $d = 4$

51st term $= a + 50d$

$= 5 + 50(4) = 205$