Jan 05

5 The first three terms of a geometric progression are 4, 2, 1.

Find the twentieth term, expressing your answer as a power of 2.

Find also the sum to infinity of this progression. [5]

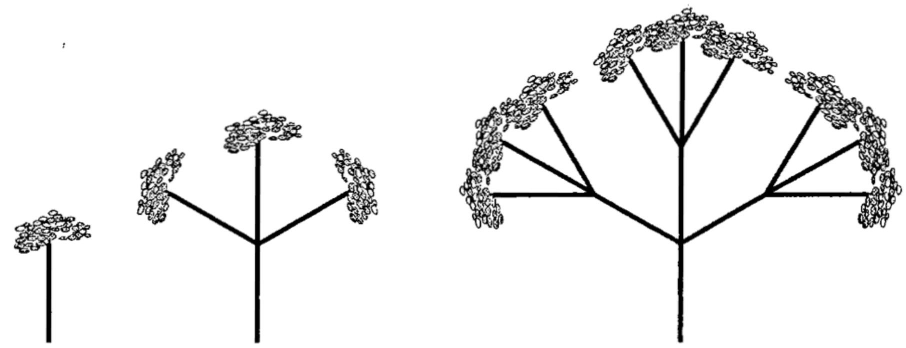
GP $a = 1, \quad r = \frac{1}{2}$

$$20^{th} \text{ term} = ar^{19} = 4 \times \left(\tfrac{1}{2}\right)^{19}$$
$$= 2^2 \times 2^{-19}$$
$$= 2^{-17}$$

$$S_\infty = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$

Jun 05

11 There is a flowerhead at the end of each stem of an oleander plant. The next year, each flowerhead is replaced by three stems and flowerheads, as shown in Fig. 11.



Year 1 Year 2 Year 3

Fig. 11

(i) How many flowerheads are there in year 5? [1]

(ii) How many flowerheads are there in year n? [1]

(iii) As shown in Fig. 11, the total number of stems in year 2 is 4, (that is, 1 old one and 3 new ones). Similarly, the total number of stems in year 3 is 13, (that is, $1 + 3 + 9$).

Show that the total number of stems in year n is given by $\frac{3^n - 1}{2}$. [2]

(iv) Kitty's oleander has a total of 364 stems. Find

(*A*) its age, [2]

(*B*) how many flowerheads it has. [1]

i) GP $a = 1, \quad r = 3$

5^{th} term $= ar^4 = 1 \times 3^4 = 81$

ii) n^{th} term $= ar^{n-1} = 1 \times 3^{n-1} = 3^{n-1}$

iii) $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(3^n - 1)}{3 - 1} = \frac{3^n - 1}{2}$

iv) A) n^{th} year $S_n = \frac{3^n - 1}{2} = 364$

$3^n - 1 = 364 \times 2$

$3^n = 364 \times 2 + 1$

$3^n = 729$

(Trial and error $3^4 = 81$, $3^5 = 243$, $3^6 = 729$)

$n = 6$

so in it's sixth year

B) $n = 6$ $ar^{n-1} = 1 \times 3^5$

$= 243$ flower heads

Jun 06

2 The first term of a geometric series is 8. The sum to infinity of the series is 10.

Find the common ratio. [3]

$S_\infty = \frac{a}{1-r}$

$10 = \frac{8}{1-r}$

$10(1-r) = 8$

$10 - 10r = 8$

$10 - 8 = 10r$

$2 = 10r$

$\frac{2}{10} = r$

$r = \frac{1}{5}$

Jan 07

2 A geometric progression has 6 as its first term. Its sum to infinity is 5.

Calculate its common ratio. [3]

$a = 6, \quad S_\infty = 5 \qquad S_\infty = \frac{a}{1-r}$

$$5 = \frac{6}{1-r}$$

$$5(1-r) = 6$$

$$5 - 5r = 6$$

$$5 - 6 = 5r$$

$$-1 = 5r$$

$$-\frac{1}{5} = r$$

$r = -\frac{1}{5}$

Jun 07

(b) In another game, played with an ordinary fair die and counters, Betty needs to throw a six to start.

The probability P_n of Betty starting on her nth throw is given by

$$P_n = \frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}.$$

(i) Calculate P_4. Give your answer as a fraction. [2]

(ii) The values $P_1, P_2, P_3, \ldots$ form an infinite geometric progression. State the first term and the common ratio of this progression.

Hence show that $P_1 + P_2 + P_3 + \ldots = 1$. [3]

i) $P_4 = \frac{1}{6} \times \left(\frac{5}{6}\right)^3 = \frac{1}{6} \times \frac{125}{216} = \frac{125}{1296}$

ii) $P_1 = a = \frac{1}{6} \times \left(\frac{5}{6}\right)^0 = \frac{1}{6} \times 1 = \frac{1}{6}$

$a = \frac{1}{6}, \quad r = \frac{5}{6}$

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{6}}{1-\frac{5}{6}} = \frac{\frac{1}{6}}{\frac{1}{6}} = 1$$

Jan 08

8 The second term of a geometric progression is 18 and the fourth term is 2. The common ratio is positive. Find the sum to infinity of this progression. [5]

2nd term $ar = 18$ ①

4th term $ar^3 = 2$ ②

② ÷ ① $\frac{ar^3}{ar} = \frac{2}{18}$

$r^2 = \frac{1}{9}$

$r = \pm\sqrt{\frac{1}{9}}$

$r = \pm\frac{1}{3}$

$r = \frac{1}{3}$ since told r is positive

Sub for r in ① $a \times \frac{1}{3} = 18$

$a = 54$

$$S_\infty = \frac{a}{1-r} = \frac{54}{1-\frac{1}{3}} = \frac{54}{2/3} = 81$$

Jun 08

2 The first term of a geometric series is 5.4 and the common ratio is 0.1.

(i) Find the fourth term of the series. [1]

(ii) Find the sum to infinity of the series. [2]

GP $a = 5.4$ $r = 0.1$

i) 4^{th} term $= ar^3 = 5.4 \times 0.1^3$

$= 0.0054$

ii) $S_\infty = \frac{a}{1-r} = \frac{5.4}{1-0.1} = \frac{5.4}{0.9} = 6$

Jan 09

8 The terms of a sequence are given by

$$u_1 = 192,$$
$$u_{n+1} = -\tfrac{1}{2}u_n.$$

(i) Find the third term of this sequence and state what type of sequence it is. **[2]**

(ii) Show that the series $u_1 + u_2 + u_3 + \ldots$ converges and find its sum to infinity. **[3]**

i) $U_1 = 192$, $U_2 = -\frac{1}{2}U_1$

$U_2 = -\frac{1}{2} \times 192$

$U_2 = -96$

$U_3 = -\frac{1}{2}U_2$

$U_3 = -\frac{1}{2}(-96)$

$U_3 = 48$

$U_1 = 192$, $U_2 = -96$, $U_3 = 48$

This is a GP with $a = 192$, $r = -\frac{1}{2}$

$$S_\infty = \frac{a}{1-r} = \frac{192}{1--\frac{1}{2}} = \frac{192}{\frac{3}{2}}$$

$$S_\infty = 128$$

Jun 09

(ii) In a 'Double Your Money' quiz game, contestants get £5 for answering the first question correctly, then a further £10 for the second question, then a further £20 for the third, and so on doubling the amount for each question until they get a question wrong and are out of the game.

(*A*) Gary received £75 from the game. How many questions did he get right? **[1]**

(*B*) Bethan answered 9 questions correctly. How much did she receive from the game? **[2]**

(*C*) State a formula for the total amount received by a contestant who answers n questions correctly. [1]

A) $5 + 10 + 20 + 40 = 75$

Got 4 questions correct

B) GP $a = 5$, $r = 2$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_9 = \frac{5(2^9 - 1)}{2-1} = £2555$$

C) $$S_n = \frac{5(2^n - 1)}{2-1} = 5(2^n - 1)$$

Jan 10

(ii) Find the sum to infinity of the geometric progression which begins

$$5 \qquad 2 \qquad 0.8 \qquad \dots .$$ **[2]**

$a = 5, \quad r = \frac{2}{5}$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{5}{1-\frac{2}{5}}$$

$$S_\infty = \frac{5}{\frac{3}{5}}$$

$$S_\infty = \frac{25}{3}$$