ARITHMETIC PROGRESSIONS

TRANSCRIPT

Example

7, 10, 13, 16, 19, 22, 25, ...

Characterised by:

1st term: 7

Common difference: 3

In general:

1st term	a
2nd term	$a + d$
3rd term	$a + 2d$
4th term	$a + 3d$
...	
n^{th} term	$a + (n-1)d$

Finding the sum of the first n terms:

Consider:

1, 2, 3, 4, 5, 6, 7, 8, ...

$$S_8 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

$$S_8 = 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

Adding

$$2S_8 = 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9$$

$$2S_8 = 8 \times 9 = 72$$

$$\therefore \quad S_8 = 72 \div 2 = 36$$

ARITHMETIC PROGRESSIONS — TRANSCRIPT

Finding the sum of the first n terms:

Consider:

$$a,\ a+d,\ a+2d,\ a+3d,\ \ldots,\ a+(n-2d),\ a+(n-1)d,..$$

$$S_n = a + a+d + a+2d + \ldots + a+(n-2)d + a+(n-1)d$$

$$S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d + \ldots + a+d + a$$

Adding

$$2S_n = 2a+(n-1)d + 2a+(n-1)d + 2a+(n-1)d + \ldots + 2a+(n-1)d + 2a+(n-1)d$$

$$2S_n = n\left(2a+(n-1)d\right)$$

$$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$$

Arithmetic Progression (AP) Summary:

1st term = a

Common difference = d

n^{th} term = $a+(n-1)d$

$$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$$

or $$S_n = \frac{n}{2}\left(a+L\right)$$

where L is the last (n^{th}) term

so $$S_n = \frac{n}{2}\left(a + a+(n-1)d\right)$$

ARITHMETIC PROGRESSIONS

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Example 1

The 4th term of an AP is 16 and the 7th term is 25. Find the first term a, the common difference d, and the sum of the first 10 terms S_{10}

4^{th} term $\quad a + 3d = 16 \quad (1)$

7^{th} term $\quad a + 6d = 25 \quad (2)$

$(2) - (1) \quad 3d = 9$

$\Rightarrow \underline{d = 3}$

Subst for d in (1)

$a + 3(3) = 16$

$a + 9 = 16$

$a = 16 - 9$

$\underline{a = 7}$

So $\quad a = 7$ and $d = 3$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\Rightarrow S_{10} = \frac{10}{2}\left(14 + 3(9)\right)$$

$$S_{10} = 5 \times 41$$

$$\underline{S_{10} = 205}$$

Example 2

Which term of the sequence 15, 18, 21, 24, 27, 30, ... is equal to 4152 ?

$a = 15, \quad d = 3$

Let n^{th} term $= 4152$

$\Rightarrow \quad a + (n-1)d = 4152$

$\Rightarrow \quad 15 + 3(n-1) = 4152$

$\Rightarrow \quad 3(n-1) = 4152 - 15$

$\Rightarrow \quad 3(n-1) = 4137$

$\Rightarrow \quad n-1 = \frac{4137}{3}$

$\Rightarrow \quad n-1 = 1379$

$\Rightarrow \quad n = 1379 + 1$

$\Rightarrow \quad n = 1380$

$\therefore$ the 1380^{th} term $= 4152$

ARITHMETIC PROGRESSIONS TRANSCRIPT

Example 3

Which term of the sequence 152, 145, 138, 124, ...
is the first to have a negative value?

$a = 152, \quad d = -7$

Let n^{th} term be first that is < 0

$$a + (n-1)d < 0$$
$$152 - 7(n-1) < 0$$
$$152 - 7n + 7 < 0$$
$$152 + 7 < 7n$$
$$159 < 7n$$
$$7n > 159$$
$$n > \frac{159}{7}$$
$$n > 22\tfrac{5}{7}$$
$$n = 23$$

23rd term is first negative term

Though not asked for in question, 23rd term would be

$$152 - 7(23-1) = 152 - 7 \times 22 = 152 - 154 = -2$$

Example 4

Granny gives John £10 on his 1st birthday and on each birthday she gives him £5 more than the previous birthday. On which birthday does the total amount John has received since birth exceed £250?

$a = 10$, $d = 5$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

We require

$$\frac{n}{2}\left(2a + (n-1)d\right) > 250$$

$$\frac{n}{2}\left(20 + 5(n-1)\right) > 250$$

$$n\left(20 + 5n - 5\right) > 500$$

$$n\left(5n + 15\right) > 500$$

$$5n^2 + 15n - 500 > 0$$

$$n^2 + 3n - 100 > 0$$

First solve

$$n^2 + 3n - 100 = 0$$

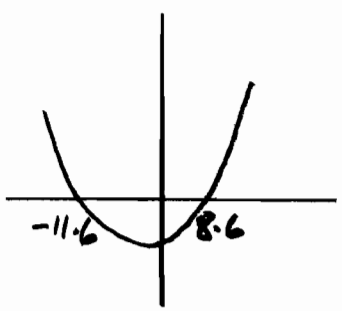
$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-3 \pm \sqrt{9 + 400}}{2}$$

$$n = \frac{-3 \pm \sqrt{409}}{2}$$

$$n \approx 8.6 \text{ or } -11.6$$

Graph of $y = n^2 + 3n - 100$



For $n^2 + 3n - 100 > 0$

$n > 8.6$

$\Rightarrow n = 9$

for smallest value of n

£250 is exceeded on John's 9th birthday